Basic modal logic

The language of basic modal logic is that of propositional logic with two extra connectives, $\Box$ and $\Diamond$. Like negation ($\neg$), they are *unary* connectives as they apply themselves to a single formula only. As done in Chapters 1 and 3, we write $p, q, r, p_3 \ldots$ to denote atomic formulas.

Definition 5.1 The formulas of basic modal logic ϕ are defined by the following Backus Naur form (BNF):

$$\phi ::= \bot \mid \top \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \to \phi) \mid (\phi \leftrightarrow \phi) \mid (\Box\phi) \mid (\Diamond\phi) \tag{5.1}$$

where p is any atomic formula.

Example formulas of basic modal logic are $(p \wedge \Diamond(p \to \Box\neg r))$ and $\Box((\Diamond q \wedge \neg r) \to \Box p)$, having the parse trees shown in Figure 5.1. The following strings are *not* formulas, because they cannot be constructed using the grammar in (5.1): $(p\Box \to q)$ and $(p \to \Diamond(q \Diamond r))$.

This convention allows us to remove many sets of brackets, retaining them only to avoid ambiguity, or to override these binding priorities.